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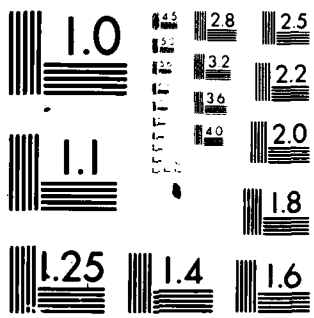

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"MICROSTRIP DIPOLES ON CYLINDRICAL STRUCTURES"

by

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N.K. Usunoglu

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An electric dipole tangent to the outer surface of a dielectric layer which coats a metallic cylinder is considered. Exact expressions are obtained for the electromagnetic field produced by the dipole, both inside the coating layer and in the surrounding free space. Asymptotic results are derived for a cylinder whose diameter is large compared to the wavelength. Arrays of elementary dipoles are discussed.

Microstrip Dipoles on Cylindrical Structures(*)

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1. Abstract

An electric dipole tangent to the outer surface of a dielectric layer which coats a metallic cylinder is considered. Exact expressions are obtained for the electromagnetic field produced by the dipole, both inside the coating layer and in the surrounding free space. Asymptotic results are derived for a cylinder whose diameter is large compared to the wavelength. Arrays of elementary dipoles are discussed.

2. Introduction

Microstrip antennas and arrays have received increasing attention in the scientific literature during the past few years, largely as a consequence of advances in printed circuit technology. The state of the art, in both theoretical and experimental studies, is summarized in the book by Bahl and Bhartia [1] and in the special issue [2], which contains two exhaustive review papers on these subjects [3,4]. The geometries of microstrip antennas are not conducive to easy analytical treatment; for example, rectangular and triangular microstrip patch antennas may be studied by combining function-theoretic methods with ray-tracing techniques [5,6,7]. Therefore numerical treatments, based e.g. on the moment method [8], have been extensively adapted, especially for computation of input impedance and mutual impedance [9,10].

Although most studies carried out so far have dealt with planar substrates, from a practical viewpoint it is very important to consider the case of printed antennas and arrays on curved surfaces, especially on portions of cylinders, cones or spheres. Together with a companion work on spherical structures [11], this paper presents a detailed study of dipoles on a dielectric-coated cylindrical structure. The exact field produced by an electric dipole tangent to the outer surface of

the coating layer is given in Section 3; the results are specialized to the radiated far field and to the surface field. An asymptotic analysis for the radiated equatorial field due to a longitudinal dipole is given in Section 4, for a thin substrate and a cylinder whose diameter is large compared to the wavelength. Preliminary results for arrays of such longitudinal dipoles are presented in Section 5. The time-dependence factor $\exp(-i\omega t)$ is omitted throughout.

3. Exact Solution

Consider an infinitely long, perfectly conducting cylinder of radius $\rho = a$ coated by a uniform layer of constant thickness $D = b - a$, permittivity $\epsilon\epsilon_o$ and permeability μ_o, and immersed in free space (see fig. 1).

Let us introduce a cylindrical coordinate system ρ,ϕ,z with the z axis on the axis of the cylinder. The primary source is an electric dipole located at $\underline{r}_o \equiv (\rho_o,\phi_o,z_o)$ where $\rho_o \geq b$, and whose electric dipole moment is

$$\underline{p} = \frac{4\pi\epsilon_o}{k}\hat{c} , \tag{1}$$

where $\hat{c}$ is a unit vector and $k = \omega\sqrt{\epsilon_o\mu_o}$ is the free-space wavenumber. The source strength of Eq. (1) corresponds to an incident (or primary) electric Hertz vector

$$\underline{\Pi}_e = \frac{e^{ikR}}{kR}\hat{c} , \; R = |\underline{r} - \underline{r}_o| , \tag{2}$$

where $\underline{r} \equiv (\rho,\phi,z)$ is the position of the observation point. It should be noted that with the primary source normalized as in Eqs. (1,2), the electric dyadic Green's function has dimensions of m^{-1}, whereas the

field is measured in m^{-2}.

The total (incident plus scattered) electric field is given by:

$$\begin{aligned}\underline{E}(\underline{r}) &= 4\pi k \underline{\underline{G}}_e^{(I)}(\underline{r};\underline{r}_o)\cdot\hat{c}\,, \qquad a \le \rho \le b\\ &= 4\pi k \underline{\underline{G}}_e^{(II)}(\underline{r};\underline{r}_o)\cdot\hat{c}\,, \qquad \rho \ge b.\end{aligned} \tag{3}$$

The electric dyadic Green's functions $\underline{\underline{G}}_e^{(I)}$ in the coating layer and $\underline{\underline{G}}_e^{(II)}$ in the surrounding medium may be obtained by the method described by Tai [12], as amended in [13,14]. It should be noted that disagreements on the singular term which appears in Eq. (6) below (see e.g. [15]) are of no relevance here, because the $\hat{\rho}\hat{\rho}$ term does not contribute to the field generated by a dipole tangent to the cylinder (i.e., $\hat{c}\cdot\hat{\rho} = 0$).

After imposing the boundary conditions at the perfectly conducting surface $\rho = a$ and across the dielectric interface $\rho = b$, as well as the radiation condition at $\rho \to \infty$, it is found that:

$$\begin{aligned}\underline{\underline{G}}_e^{(I)}(\underline{r};\underline{r}_o) = \frac{i}{8\pi}\int_{-\infty}^{\infty} du \sum_{n=0}^{\infty}\frac{\tau_n}{\eta^2}\sum_{e,o}\Bigg(\Bigg\{ & A_n\left[\underline{M}^{(1)}_{\substack{e\\o}n,\xi}(u,\underline{r}) + \alpha_n \underline{M}^{(3)}_{\substack{e\\o}n,\xi}(u,\underline{r})\right] \mp \\ & \mp C_n\left[\underline{N}^{(1)}_{\substack{o\\e}n,\xi}(u,\underline{r}) + \beta_n \underline{N}^{(3)}_{\substack{o\\e}n,\xi}(u,\underline{r})\right]\Bigg\}\underline{M}^{(3)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o) + \\ & + \Bigg\{B_n\left[\underline{N}^{(1)}_{\substack{e\\o}n,\xi}(u,\underline{r}) + \beta_n \underline{N}^{(3)}_{\substack{e\\o}n,\xi}(u,\underline{r})\right] \mp \\ & \mp D_n\left[\underline{M}^{(1)}_{\substack{o\\e}n,\xi}(u,\underline{r}) + \alpha_n \underline{M}^{(3)}_{\substack{o\\e}n,\xi}(u,\underline{r})\right]\Bigg\}\underline{N}^{(3)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o)\Bigg)\,,\end{aligned} \tag{4}$$

$$\underline{\underline{G}}_e^{(II)}(\underline{r};\underline{r}_o) = \underline{\underline{G}}_{eo}(\underline{r};\underline{r}_o) + \underline{\underline{G}}_{es}(\underline{r};\underline{r}_o)\,, \tag{5}$$

$$\underline{\underline{G}}_{eo}(\underline{r};r_o) = -k^{-2}\hat{\rho}\hat{\rho}\,\delta(\underline{r}-\underline{r}_o) + \underline{\underline{G}}_{eo}^{(\gtrless)}(\underline{r};\underline{r}_o), \quad (\rho \gtrless \rho_o)\,, \tag{6}$$

$$\begin{aligned}\underline{\underline{G}}_{eo}^{(>)}(\underline{r};\underline{r}_o) = \frac{i}{8\pi}\int_{-\infty}^{\infty} du \sum_{n=0}^{\infty}\frac{\tau_n}{\eta^2}\sum_{e,o}\Big[& \underline{M}^{(3)}_{\substack{e\\o}n,\eta}(u,\underline{r})\underline{M}^{(1)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o) + \\ & + \underline{N}^{(3)}_{\substack{e\\o}n,\eta}(u,\underline{r})\underline{N}^{(1)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o)\Big]\,, \quad (\rho > \rho_o)\,,\end{aligned} \tag{7}$$

$$\underline{\underline{G}}_{eo}^{(<)}(\underline{r};\underline{r}_o) = \frac{i}{8\pi}\int_{-\infty}^{\infty} du \sum_{n=0}^{\infty} \frac{\tau_n}{\eta^2} \sum_{e,o} \Bigg[\underline{M}^{(1)}_{\substack{e\\o}n,\eta}(u,\underline{r})\underline{M}^{(3)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o) + \tag{8}$$

$$+ \underline{N}^{(1)}_{\substack{e\\o}n,\eta}(u,\underline{r})\underline{N}^{(3)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o)\Bigg] , \quad (\rho < \rho_o),$$

$$\underline{\underline{G}}_{es}(\underline{r};\underline{r}_o) = \frac{i}{8\pi}\int_{-\infty}^{\infty} du \sum_{n=0}^{\infty} \frac{\tau_n}{\eta^2} \sum_{e,o} \Bigg\{ \Big[a_n \underline{M}^{(3)}_{\substack{e\\o}n,\eta}(u,\underline{r}) \mp$$

$$\mp c_n \underline{N}^{(3)}_{\substack{o\\e}n,\eta}(u,\underline{r})\Big] \underline{M}^{(3)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o) + \Big[b_n \underline{N}^{(3)}_{\substack{e\\o}n,\eta}(u,\underline{r}) \mp \tag{9}$$

$$\mp c_n \underline{M}^{(3)}_{\substack{o\\e}n,\eta}(u,\underline{r})\Big] \underline{N}^{(3)}_{\substack{e\\o}n,\eta}(-u,\underline{r}_o)\Bigg\} ,$$

where

$$\underline{M}^{(j)}_{\substack{e\\o}n,\eta}(u,\underline{r}) = \nabla \times \left[Z_n^{(j)}(\eta\rho)\begin{matrix}\cos\\ \sin\end{matrix}(n\phi) e^{iuz}\hat{z}\right] = \tag{10}$$

$$= e^{iuz}\left[\mp \frac{n}{\rho} Z_n^{(j)}(\eta\rho)\begin{matrix}\sin\\ \cos\end{matrix}(n\phi)\hat{\rho} - \frac{\partial}{\partial\rho} Z_n^{(j)}(\eta\rho)\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{\phi}\right] ,$$

$$\underline{N}^{(j)}_{\substack{e\\o}n,\eta}(u,\underline{r}) = \frac{1}{\sqrt{u^2+\eta^2}} \nabla \times \underline{M}^{(j)}_{\substack{e\\o}n,\eta}(u,\underline{r}) = \tag{11}$$

$$= \frac{e^{iuz}}{\sqrt{u^2+\eta^2}} \left[iu \frac{\partial}{\partial\rho} Z_n^{(j)}(\eta\rho)\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{\rho} \mp \frac{inu}{\rho} Z_n^{(j)}(\eta\rho)\begin{matrix}\sin\\ \cos\end{matrix}(n\phi)\hat{\phi} + \right.$$

$$\left. + \eta^2 Z_n^{(j)}(\eta\rho)\begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\hat{z}\right] ,$$

$$\eta = \sqrt{k^2 - u^2} \;, \quad \xi = \sqrt{k_1^2 - u^2} \;, \quad k_1 = \epsilon k = N^2 k \;, \tag{12}$$

$j = 1$ or 3, $Z_n^{(1)}(x) = J_n(x)$ and $Z_n^{(3)}(x) = H_n^{(1)}(x)$ are the Bessel function and the Hankel function of the first kind, $\delta(\underline{r} - \underline{r}_o)$ is the three-dimensional Dirac delta-function, $\tau_o = 1$ and $\tau_{n>0} = 2$, and

the integral path along the real u-axis passes below the points $u = k$, $u = k_1$ and above the points $u = -k$, $u = -k_1$ (see fig. 2).

The various coefficients which appear in Eqs. (4) and (9) are given by (the prime means derivative with respect to the argument of the primed function):

$$\alpha_n = -\frac{J_n'(\xi a)}{H_b^{(1)}\gamma(\xi a)} \quad , \quad \beta_n = -\frac{J_n(\xi a)}{H_n^{(1)}(\xi a)} \tag{13}$$

$$A_n = N\frac{\Gamma_{n\beta}}{\Gamma_{n\alpha}} B_n = \frac{2i\Gamma_{n\beta}}{\pi b\delta_n H_n^{(1)}(\eta b)} , \tag{14}$$

$$C_n = N\frac{\gamma_{n\alpha}}{\gamma_{n\beta}} D_n = \frac{2nu\gamma_{n\alpha}}{\pi k_1 b^2 \delta_n H_n^{(1)}(\eta b)} \left(1 - \frac{\xi^2}{\eta^2}\right) , \tag{15}$$

$$a_n = -\frac{J_n(\eta b)}{H_n^{(1)}(\eta b)} + \frac{\xi^2\gamma_{n\alpha}}{\eta^2 H_n^{(1)}(\eta b)} A_n , \tag{16}$$

$$b_n = -\frac{J_n(\eta b)}{H_n^{(1)}(\eta b)} + \frac{\xi^2\gamma_{n\beta}}{N\eta^2 H_n^{(1)}(\eta b)} B_n , \tag{17}$$

$$c_n = \frac{\xi^2\gamma_{n\beta}}{N\eta^2 H_n^{(1)}(\eta b)} C_n , \tag{18}$$

where

$$\gamma_{n\alpha} = J_n(\xi b) + \alpha_n H_n^{(1)}(\xi b), \quad \gamma_{n\beta} = J_n(\xi b) + \beta_n H_n^{(1)}(\xi b) , \tag{19}$$

$$\Gamma_{n\alpha} = -\frac{\partial\gamma_{n\alpha}}{\partial b} + \frac{\xi^2}{\eta^2}\gamma_{n\alpha}\frac{\partial}{\partial b}\ln H_n^{(1)}(\eta b) , \tag{20}$$

$$\Gamma_{n\beta} = -\frac{\partial\gamma_{n\beta}}{\partial b} + \frac{\xi^2}{\epsilon\eta^2}\gamma_{n\beta}\frac{\partial}{\partial b}\ln H_n^{(1)}(\eta b) , \tag{21}$$

$$\delta_n = \Gamma_{n\alpha}\Gamma_{n\beta} - \left(\frac{nu}{k_1 b}\right)^2 \left(1 - \frac{\xi^2}{\eta^2}\right)^2 \gamma_{n\alpha}\gamma_{n\beta}. \tag{22}$$

The above formulas can be considerably simplified in particular case. Consider, for example, an axially-oriented dipole $(\hat{c} = \hat{z})$ located on the substrate $(\rho_o = b)$; the total electric field on the substrate and in the equatorial plane $z = z_o$ is:

$$[\underline{E}]_{\substack{\rho=\rho_o=b \\ z=z_o \\ \hat{c}=\hat{z}}} = \hat{z}\,\frac{-2}{\pi\epsilon k^2 b}\sum_{n=0}^{\infty} \tau_n \cos n(\phi - \phi_o)\int_0^{\infty} du\,\gamma_{n\beta}\Gamma_{n\alpha}\xi^2\delta_n^{-1}. \tag{23}$$

In the more general case when the dipole at $\underline{r}_o = (b,\phi_o,z_o)$ is tangent to the substrate but not necessarily axially oriented, i.e.

$$\hat{c} = \hat{c}\cdot\hat{\phi}_o\hat{\phi}_o + \hat{c}\cdot\hat{z}\hat{z}\,, \quad \hat{c}\cdot\hat{\rho}_o = 0\,, \tag{24}$$

then the electric field at any point $\underline{r}$ in free space is:

$$\underline{E}(\underline{r}) = \underline{E}_{\perp}\,\hat{c}\cdot\hat{\phi}_o + \underline{E}_{\parallel}\,\hat{c}\cdot\hat{z}\,, \tag{25}$$

where

$$\underline{E}_{\perp} = \frac{1}{\pi k b}\int_{-\infty}^{\infty} du\, k^2\eta^{-2} e^{-iuz_o}\sum_{n=0}^{\infty}\frac{\tau_n}{H_n^{(1)}(\eta b)}\sum_{e,o}\begin{matrix}\cos\\ \sin\end{matrix}(n\phi_o)\,\cdot$$

$$\cdot\left\{\left[-1 + \frac{\xi^2\gamma_{n\alpha}\Gamma_{n\beta}H_n^{(1)\prime}(\eta b)}{\eta\delta_n H_n^{(1)}(\eta b)} - \frac{n^2u^2\xi^2\gamma_{n\alpha}\gamma_{n\beta}}{\eta^2 k_1^2 b^2\delta_n}\left(\frac{\xi^2}{\eta^2} - 1\right)\right]\underline{M}^{(3)}_{\substack{e\\o}n,\eta}(u,\underline{r}) \pm\right.$$

$$\left.\pm\,\frac{inu\gamma_{n\beta}\xi^2}{\epsilon k b\delta_n\eta}\left[\eta^{-1}\Gamma_{n\alpha} - \gamma_{n\alpha}\left(\frac{\xi^2}{\eta^2} - 1\right)\frac{H_n^{(1)\prime}(\eta b)}{H_n^{(1)}(\eta b)}\right]\underline{N}^{(3)}_{\substack{o\\e}n,\eta}(u,\underline{r})\right\}, \tag{26}$$

$$\underline{E}_{\parallel} = \frac{1}{\pi\epsilon b}\int_{-\infty}^{\infty} du\,\xi^2\eta^{-2} e^{-iuz_o}\sum_{n=0}^{\infty}\frac{\tau_n\gamma_{n\beta}}{\delta_n H_n^{(1)}(\eta b)}\sum_{e,o}\begin{matrix}\cos\\ \sin\end{matrix}(n\phi_o)\,\cdot$$

$$\cdot\left[-\Gamma_{n\alpha}\,\underline{N}^{(3)}_{\substack{e\\o}n,\eta}(u,\underline{r}) \pm \frac{inu}{kb}\left(\frac{\xi^2}{\eta^2}-1\right)\gamma_{n\alpha}\,\underline{M}^{(3)}_{\substack{o\\e}n,\eta}(u,\underline{r})\right]. \tag{27}$$

In the far field $(\rho \to \infty)$, the integrals in Eqs. (26-27) may be asymptotically evaluated by the method of stationary phase, the stationary point being at $u = k\cos\theta$, where $\theta = \arccos(z/r)$ is the usual polar angle in spherical coordinates. If we write the radiated field as

$$\left.\begin{aligned} \underline{E}_{\parallel}(\underline{r}) &= -k^2\,\frac{e^{ikr}}{kr}\,\underline{S}_{\parallel}(\hat{r}), \\ \underline{E}_{\perp}(\underline{r}) &= -k^2\,\frac{e^{ikr}}{kr}\,\underline{S}_{\perp}(\hat{r}), \end{aligned}\right\} (\rho \to \infty) \tag{28}$$

then the dimensionless far-field coefficients $\underline{S}_{\parallel}$ and $\underline{S}_{\perp}$ are

$$\begin{aligned} \underline{S}_{\parallel}(\hat{r}) &= S_{\parallel\theta}\hat{\theta} + S_{\parallel\phi}\hat{\phi} \\ &= \frac{2i}{\pi\zeta}\left(1-\frac{\cos^2\theta}{\varepsilon}\right)e^{-ikz_o\cos\theta}(A_\theta\hat{\theta} + A_\phi\hat{\phi}), \end{aligned} \tag{29}$$

$$\begin{aligned} \underline{S}_{\perp}(\hat{r}) &= S_{\perp\theta}\hat{\theta} + S_{\perp\phi}\hat{\phi} \\ &= \frac{2}{\pi\zeta}\,e^{-ikz_o\cos\theta}(B_\theta\hat{\theta} + B_\phi\hat{\phi}), \end{aligned} \tag{30}$$

where

$$\zeta = kb\sin\theta,$$

$$A_\theta = \sum_{n=0}^{\infty}\tau_n(-i)^n\,\frac{k\tilde{\gamma}_{n\beta}\tilde{\Gamma}_{n\alpha}}{\tilde{\delta}_n H_n^{(1)}(\zeta)}\cos n(\phi-\phi_o), \tag{32}$$

$$A_\phi = (\varepsilon-1)\,\frac{2\cot\theta}{\zeta}\sum_{n=1}^{\infty}(-i)^n n\,\frac{k^2\tilde{\gamma}_{n\alpha}\tilde{\gamma}_{n\beta}}{\tilde{\delta}_n H_n^{(1)}(\zeta)}\sin n(\phi-\phi_o), \tag{33}$$

$$B_\theta = \left(1 - \frac{\cos^2\theta}{\epsilon}\right) \frac{2\cot\theta}{\zeta} \sum_{n=1}^{\infty} (-i)^n n \frac{k^2\tilde{\gamma}_{n\beta}}{\tilde{\delta}_n H_n^{(1)}(\zeta)} \left[k^{-1}\tilde{\Gamma}_{n\alpha} - \frac{\epsilon - 1}{\sin\theta} \tilde{\gamma}_{n\alpha} \frac{H_n^{(1)'}(\zeta)}{H_n^{(1)}(\zeta)}\right] \sin n(\phi - \phi_o) \;, \tag{34}$$

$$B_\phi = \sum_{n=0}^{\infty} \frac{\tau_n(-i)^n}{H_n^{(1)}(\zeta)} \left[-1 + \frac{\epsilon - \cos^2\theta}{\sin\theta} \cdot \frac{k\tilde{\gamma}_{n\alpha}\tilde{\Gamma}_{n\beta}H_n^{(1)'}(\zeta)}{\tilde{\delta}_n H_n^{(1)}(\zeta)} - \frac{n^2\cot^2\theta}{\epsilon\zeta^2} (\epsilon - 1)(\epsilon - \cos^2\theta) \frac{k^2\tilde{\gamma}_{n\alpha}\tilde{\gamma}_{n\beta}}{\tilde{\delta}_n}\right] \cos n(\phi - \phi_o) \;, \tag{35}$$

and $\tilde{f} = (f)_{u=k\cos\theta}$. In particular, observe that $S_{\|\theta}$ and $S_{\|\phi}$ are even and odd functions of $(\phi - \phi_o)$, respectively, as it must be by reason of symmetry. Also, $S_{\|\phi} = 0$ when $\epsilon = 1$. If the cylindrical structure were absent, the dipole at the origin $(\rho_o = z_o = 0)$ and axially oriented $(\hat{c} = \hat{z})$ would yield $S_{\|\theta} = \sin\theta$, $S_{\|\phi} = 0$, as expected.

4. Asymptotic Expansions for Thin Substrate

We limit our considerations to the far field produced by a dipole on the substrate and parallel to the z axis, in the equatorial plane $\theta = \frac{\pi}{2}$, so that $S_{||\phi} = 0$. We assume

$$kb \gg 1, \quad |k_1 D| \ll 1 ; \tag{36}$$

the second inequality means that the coating layer is electrically thin. Then

$$\begin{aligned} Z_n(k_1 a) &\approx Z_n(k_1 b) - k_1 D Z_n'(k_1 b) , \\ Z_n'(k_1 a) &\approx Z_n'(k_1 b) + k_1 D \left[1 - \frac{n^2}{(k_1 b)^2}\right] Z_n(k_1 b), \end{aligned} \tag{37}$$

where $Z_n = J_n$ or $H_n^{(1)}$; substitution into (32) yields, for $\theta = \pi/2$:

$$(S_{||\theta})_{\theta=\frac{\pi}{2}} = S = -\frac{2ikD}{\pi kb} \sum_{n=0}^{\infty} \frac{\tau_n (-1)^n \cos n(\phi - \phi_o)}{H_n^{(1)}(kb) - kDH_n^{(1)'}(kb)} . \tag{38}$$

Set:

$$\begin{aligned} \Phi &= \phi - \phi_o - \pi \\ \eta_o &= -ikD, \end{aligned} \tag{39}$$

and observe that η_o would be the relative surface impedance of a thin substrate with magnetic permeability equal to that of free space. Then:

$$S = \frac{2\eta_o}{\pi kb} \sum_{n=-\infty}^{\infty} \frac{(-1)^n e^{-i\frac{\pi}{2}n} \cos n\phi}{H_n^{(1)}(kb) - i\eta_o H_n^{(1)'}(kb)}$$

$$= \frac{2i\eta}{\pi kb} \int_C \frac{e^{-i\frac{\pi}{2}\nu} \cos \nu\phi}{\left[H_\nu^{(1)}(kb) - i\eta_o H_\nu^{(1)'}(kb)\right] \sin \pi\nu} d\nu \qquad (40)$$

where the contour C is along the real axis and just above it in the complex ν-plane, from $-\infty$ to $+\infty$. The integral in (40) is similar to the one studied by Goriainov [16] in relation to plane-wave scattering by a cylinder. Following [16], we set

$$e^{-i\nu\frac{\pi}{2}} = e^{i\nu\frac{3\pi}{2}} - 2ie^{i\nu\frac{\pi}{2}} \sin \pi\nu \qquad (41)$$

in the integrand of Eq. (40), so that

$$S = S_1 + S_2$$

where

$$S_1 = \frac{4\eta_o}{\pi kb} \int_C \frac{e^{i\nu\frac{\pi}{2}} \cos\nu\phi}{M_\nu(kb)} d\nu , \qquad (42)$$

$$S_2 = \frac{2i\eta_o}{\pi kb} \int_C \frac{e^{i\nu\frac{3\pi}{2}} \cos\nu\phi}{M_\nu(kb) \sin\pi\nu} d\nu , \qquad (43)$$

with

$$M_\nu(kb) = H_\nu^{(1)}(kb) - i\eta_o H_\nu^{(1)'}(kb). \qquad (44)$$

The integral S_1 has a stationary point, as is seen by using Debye's expansion for $H_\nu^{(1)}$ in (44); the integral S_2 does not have a stationary point. Assume

$$|\nu - kb| > |\nu^{1/3}| , \qquad (45)$$

then, in a region about the origin in the ν-plane (for details see, e.g. [17,18]):

$$S_1 \sim \eta_o \sqrt{\frac{2}{\pi kb}} \int_C d\nu \frac{\sqrt[4]{1 - (\frac{\nu}{kb})^2}}{1 - i\eta_o\sqrt{(\frac{\nu}{kb})^2 - 1}}$$

$$\cdot \left\{ \exp i\left[\nu(\frac{\pi}{2} + \phi + \arccos\frac{\nu}{kb}) - \sqrt{(kb)^2 - \nu^2} + \frac{\pi}{4}\right] + \exp i\left[\nu(\frac{\pi}{2} - \phi + \arccos\frac{\nu}{kb}) - \sqrt{(kb)^2 - \nu^2} + \frac{\pi}{4}\right]\right\}; \tag{46}$$

the first integral in (46) has a stationary point at $\nu_o = -kb\sin\phi$, whereas the second integral has no stationary point. A stationary phase evaluation of S is therefore obtained by considering the stationary phase contribution due to the first term in the integrand of Eq. (46):

$$S \sim -\frac{2ikD\cos(\phi - \phi_o)}{1 - ikD\cos(\phi - \phi_o)} e^{-ikb\cos(\phi - \phi_o)}. \tag{47}$$

On the basis of Eq. (36 the denominator in (47) may be replaced by unity, so that

$$S \sim -2ikD\cos(\phi - \phi_o)e^{-ikb\cos(\phi - \phi_o)}. \tag{48}$$

At the point of stationary phase, condition (45) is satisfied if

$$|\phi - \phi_o| < \frac{\pi}{2} - \sqrt{2}\,(kb)^{-4/3}, \tag{49}$$

which defines the region of free space into which direct radiation by the dipole occurs. Thus, Eq. (48) is valid in the "illuminated region" defined by (49), as shown in Fig. 3.

The far field in the penumbra and shadow regions, where inequality (49) does not hold, is obtained by letting

$$\nu = kb + mt\ , \qquad m = \left(\frac{kb}{2}\right)^{1/3} \gg 1 \tag{50}$$

into Eq. (44), so that:

$$M_\nu(kb) \sim \frac{-i}{m\sqrt{\pi}}\,[w_1(t) + \frac{kD}{m}\,w_1'(t)]\ , \tag{51}$$

where $w_1(t)$ is Airy's function in Fock's notation [18]. The poles of (40) in the complex ν-plane are the zeros of $M_\nu(kb)$, i.e.:

$$\frac{w_1'(t_s)}{w_1(t_s)} = -\ \frac{m}{kD}\ . \tag{52}$$

Since this last ratio is large compared to unity,

$$t_s \simeq t_{os} - \frac{kD}{m}\ , \tag{53}$$

where the zeros $t_{os}(s = 1,2,\ldots)$ of $w_1(t_{os}) = 0$ are well tabulated.

Since $\mathrm{Im}\nu > 0$ at t_s, we may rewrite Eq. (40) as:

$$\begin{aligned} S \sim \frac{kD}{m}\Bigg[& f(\xi_+, \frac{kD}{m})\ e^{ikb(\frac{\pi}{2} + \phi)} \\ & + f(\xi_-, \frac{kD}{m})\ e^{ikb(\frac{\pi}{2} - \phi)}\Bigg], \end{aligned} \tag{54}$$

where

$$\xi_{\pm} = m(\frac{\pi}{2} \pm \phi)\ , \tag{55}$$

and the generalized Fock function

$$f(\xi,\rho) = \frac{1}{\sqrt{\pi}} \int_\Gamma \frac{e^{i\xi t}}{w_1(t) + \rho w_1'(t)}\ dt \tag{56}$$

is well known, and can be evaluated e.g. by residues at the poles (53); the contour Γ starts at infinity in the angular sector $\pi > \arg t > \pi/3$,

passes between $\nu = kb$ and the pole of the integrand nearest the origin (i.e. $t = t_1$), and ends at infinity in the angular sector $\pi/3 > \arg t > - \pi/3$ (see fig. 4).

The approximation (54) includes only the first two creeping waves, which complete less than one complete turn around the cylinder; the geometric interpretation of the two terms in Eq. (54) is shown in fig. 5.

5. Arrays of Longitudinal Dipoles

An axially oriented dipole at angular position ϕ_o on the substrate produces the far-field pattern of Eq. (48) in the illuminated portion of its equatorial plane. Consider an array of n such dipoles with angular separation α between dipoles, i.e. the total array angle is $(n - 1)\alpha$ (see fig. 6). The far-field point of observation is in the illuminated region of all dipoles if

$$-\frac{\pi}{2} + (n - 1)\alpha + \sqrt{2}(kb)^{-4/3} < \phi < \frac{\pi}{2} - \sqrt{2}(kb)^{-4/3}. \qquad (57)$$

Under limitation (57), dipoles fed with equal amplitude and progressive phase shift β yield the pattern:

$$S_{total} = 2kD \frac{\partial}{\partial(kb)} \sum_{\ell=0}^{n-1} e^{-ikb\cos(\phi - \ell\alpha) + i\ell\beta}. \qquad (58)$$

6. Concluding Remarks

The basic analysis for studying the behavior of printed circuit antennas on cylindrical structures has been presented herein. Numerical results pertaining to current distribution and other antenna characteristics for various substrate parameters will be presented.

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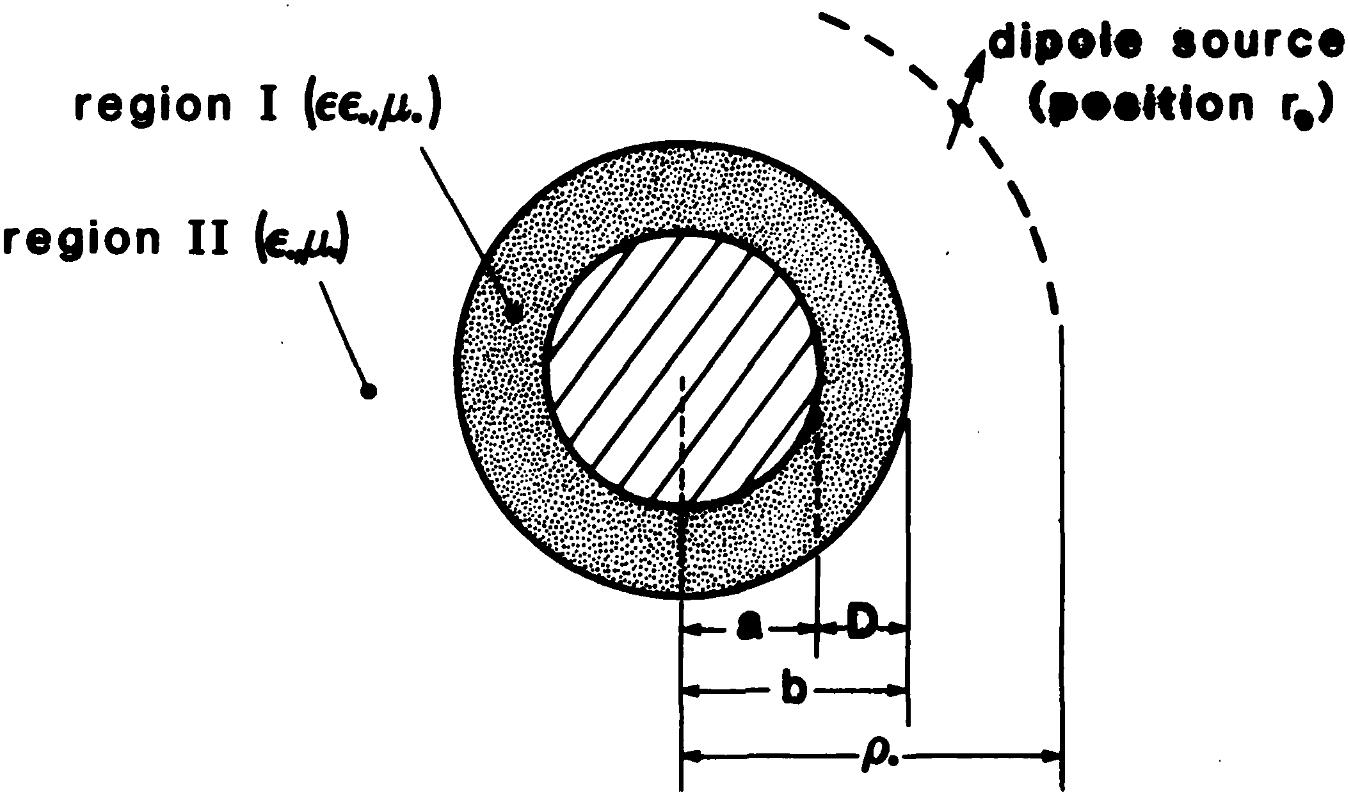


Figure 1 Geometry of the Problem

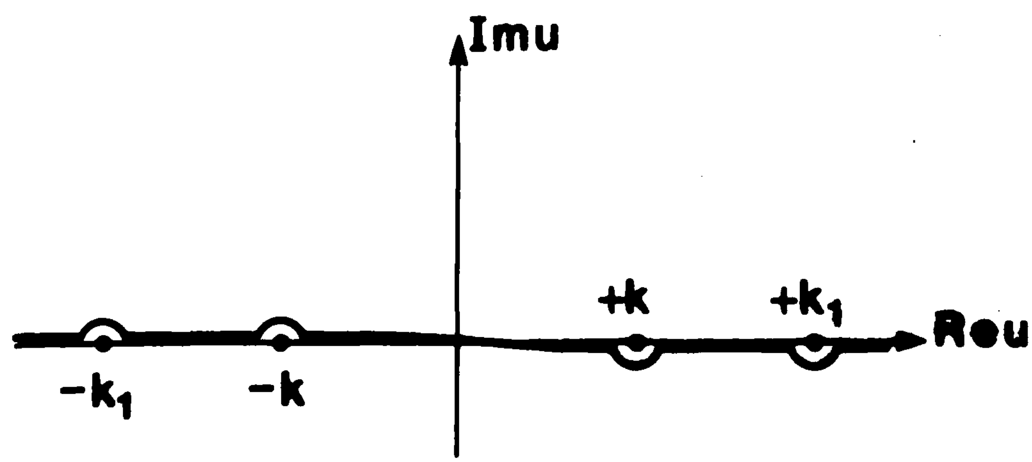


Figure 2 Path of Integration Along the Reu-axis

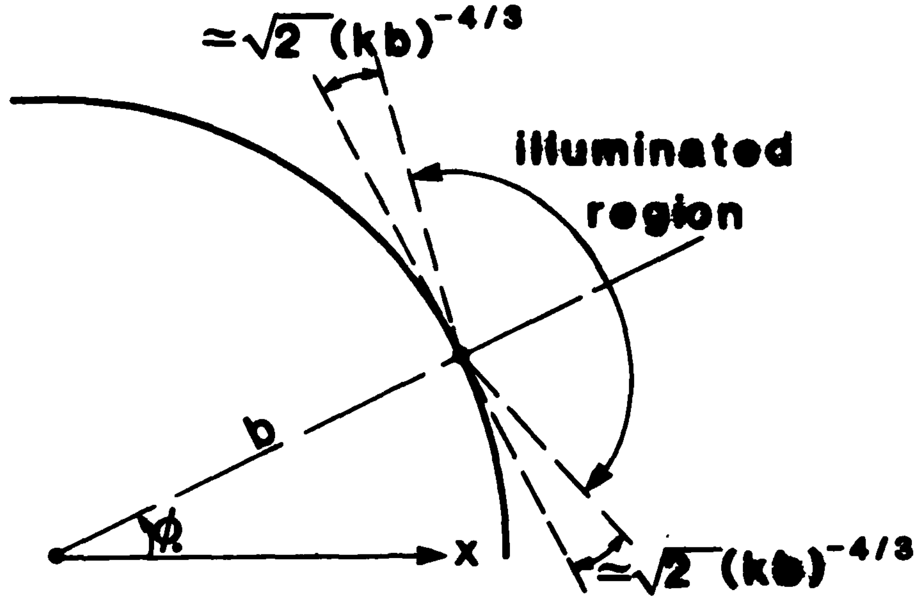


Figure 3 Geometric Interpretation of Condition (49)

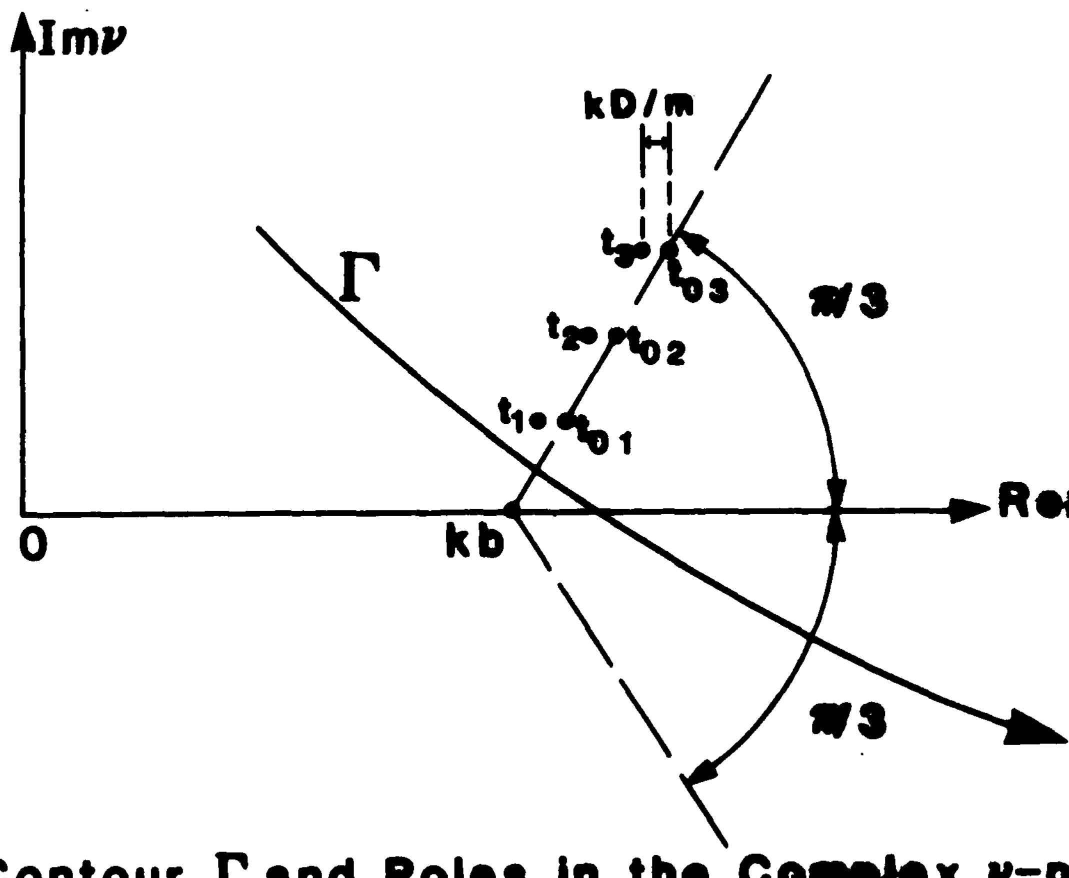
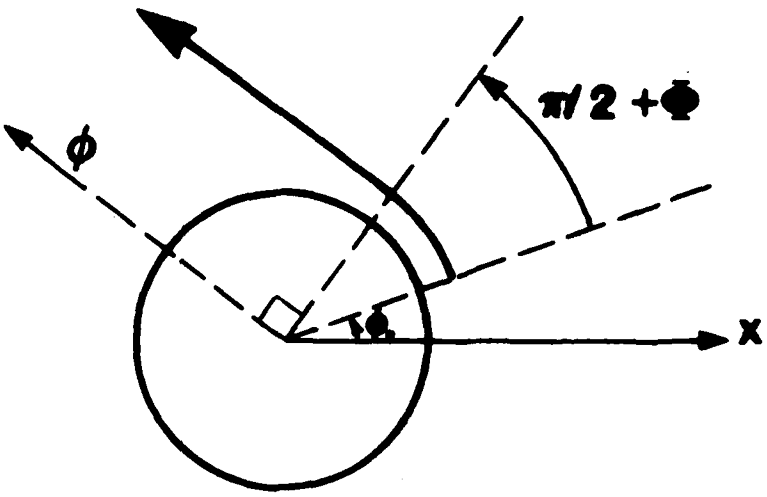
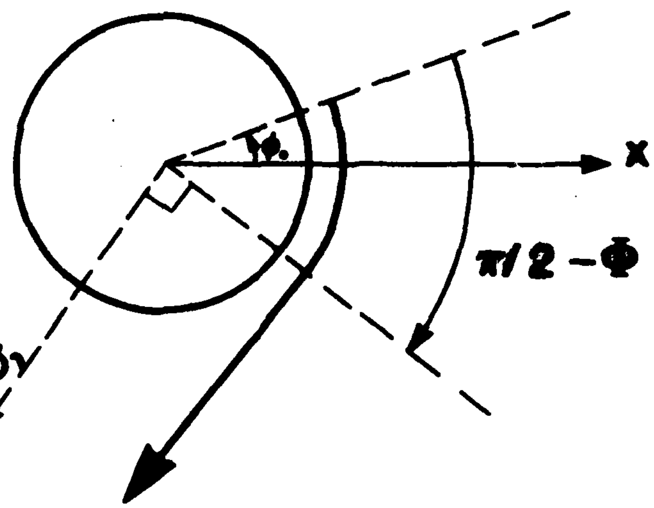


Figure 4 Contour Γ and Poles in the Complex ν-plane



(a)



(b)

Figure 5 Geometric Interpretation of the Creeping-wave Terms in Equation (54); (a) $\xi=\xi_+$; (b) $\xi=\xi_-$

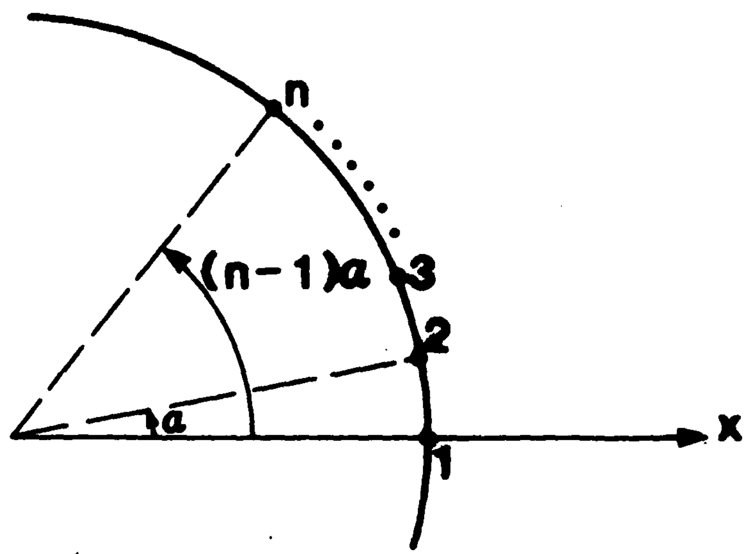


Figure 6 Geometry for Circumferential Array

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